

# ADVANCED GCE MATHEMATICS (MEI)

4763

Mechanics 3

Candidates answer on the answer booklet.

#### **OCR** supplied materials:

- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

#### Other materials required:

· Scientific or graphical calculator

## Wednesday 22 June 2011 Morning

**Duration:** 1 hour 30 minutes



#### **INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \, \text{m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

#### **INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail
  of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- This document consists of **8** pages. Any blank pages are indicated.

1 A particle is moving in a straight line. At time *t* its displacement *x* from a fixed point O on the line is given by

$$x = A \sin \omega t$$

where A and  $\omega$  are constants.

(i) Show that 
$$\frac{d^2x}{dt^2} = -\omega^2 x$$
 and  $\left(\frac{dx}{dt}\right)^2 = \omega^2 (A^2 - x^2)$ . [5]

A ball floats on the surface of the sea. Waves cause the ball to rise and fall in a vertical line, and the ball is executing simple harmonic motion. The centre of the oscillations is 8 m above the sea-bed. The ball has speed  $1.2 \text{ m s}^{-1}$  when it is 7.3 m above the sea-bed, and it has speed  $0.75 \text{ m s}^{-1}$  when it is 10 m above the sea-bed.

- (ii) Show that the amplitude of the oscillations is 2.5 m, and find the period. [6]
- (iii) Find the maximum speed of the ball. [1]
- (iv) Find the magnitude and direction of the acceleration of the ball when it is 6.4 m above the sea-bed.
- (v) Find the time taken for the ball to move upwards from 6 m above the sea-bed to 9 m above the sea-bed. [3]

- 2 (a) A particle P of mass 0.2 kg is connected to a fixed point O by a light inextensible string of length 3.2 m, and is moving in a vertical circle with centre O and radius 3.2 m. Air resistance may be neglected. When P is at the highest point of the circle, the tension in the string is 0.6 N.
  - (i) Find the speed of P when it is at the highest point. [3]
  - (ii) For an instant when OP makes an angle of  $60^{\circ}$  with the downward vertical, find
    - (A) the radial and tangential components of the acceleration of P, [5]
    - (B) the tension in the string. [3]
  - (b) A solid cone is fixed with its axis of symmetry vertical and its vertex V uppermost. The semi-vertical angle of the cone is 36°, and its surface is smooth. A particle Q of mass 0.2 kg is connected to V by a light inextensible string, and Q moves in a horizontal circle at constant speed, in contact with the surface of the cone, as shown in Fig. 2.

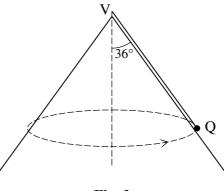


Fig. 2

The particle Q makes one complete revolution in  $1.8\,\mathrm{s}$ , and the normal reaction of the cone on Q has magnitude  $0.75\,\mathrm{N}$ .

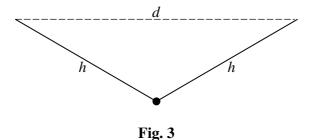
(i) Find the tension in the string. [2]

(ii) Find the length of the string. [5]

3 Fixed points A and B are 4.8 m apart on the same horizontal level. The midpoint of AB is M. A light elastic string, with natural length 3.9 m and modulus of elasticity 573.3 N, has one end attached to A and the other end attached to B.

A particle P is attached to the midpoint of the string, and is released from rest at M. It comes instantaneously to rest when P is 1.8 m vertically below M.

In general, a light elastic string, with natural length a and modulus of elasticity  $\lambda$ , has its ends attached to fixed points which are a distance d apart on the same horizontal level. A particle of mass m is attached to the midpoint of the string, and in the equilibrium position each half of the string has length h, as shown in Fig. 3.



When the particle makes small oscillations in a vertical line, the period of oscillation is given by the formula

$$\sqrt{\frac{8\pi^2h^3}{8h^3-ad^2}}\,m^{\alpha}a^{\beta}\lambda^{\gamma}.$$

(iv) Show that 
$$\frac{8\pi^2 h^3}{8h^3 - ad^2}$$
 is dimensionless. [1]

(v) Use dimensional analysis to find  $\alpha$ ,  $\beta$  and  $\gamma$ .

(vi) Hence find the period when the particle P makes small oscillations in a vertical line centred on the position of equilibrium given in part (iii). [2]

The region A is bounded by the curve  $y = x^2 + 5$  for  $0 \le x \le 3$ , the x-axis, the y-axis and the line x = 3. The region B is bounded by the curve  $y = x^2 + 5$  for  $0 \le x \le 3$ , the y-axis and the line y = 14. These regions are shown in Fig. 4.

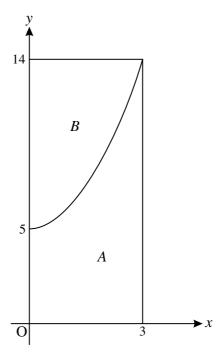


Fig. 4

- (i) Find the coordinates of the centre of mass of a uniform lamina occupying the region A. [9]
- (ii) The region B is rotated through  $2\pi$  radians about the y-axis to form a uniform solid of revolution R. Find the y-coordinate of the centre of mass of the solid R. [6]
- (iii) The region A is rotated through  $2\pi$  radians about the y-axis to form a uniform solid of revolution S. Using your answer to part (ii), or otherwise, find the y-coordinate of the centre of mass of the solid S.

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**GCE** 

# **Mathematics (MEI)**

**Advanced GCE** 

Unit 4763: Mechanics 3

# Mark Scheme for June 2011

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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#### General marking instructions for MEI Mechanics

#### Types of marks

- M A suitable method has been selected and applied in a manner which shows the method is essentially understood. Method marks are not generally lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
- A Follows M mark. Unless modified, it implies that errors already penalized in **earlier** parts of the question will be followed through for **no further loss**. It does **not** imply the following through of errors made in the **current** part of the question. A marks cannot be given unless the associated M mark is earned (including by implication).
- **B** Freestanding. Unless modified, it is **cao**.
- **E** A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
- **F** Specifies that working should be followed through within a part of a question or emphasizes that follow through is required from an earlier part of a question.

#### **General instructions**

- 1. Mark in red. Correct answers should be ticked, errors which determine marks (usually the first error in each part of a question) should be identified by ringing or by a cross or by underlining or, if an omission, by a caret <sup>A</sup>. Do not cross out or obliterate any work. In cases of particular difficulty, brief annotations that explain the marks awarded (or not awarded) are correct or indicate that calculations correctly FT may be helpful should the script be reviewed at a later stage but, in general, comments on the candidate's work or corrections to it should not be written on the script. Each page of the script which includes any part of the candidate's answer must have some indication that it has been seen, e.g. a tick.
- 2. The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt.
- 3. Unless otherwise indicated, marks once gained cannot subsequently be lost, (e.g. wrong working following a correct form of answer is ignored). However, this would not apply to a case where a candidate passed through the correct solution as part of a wrong argument, (e.g. when asked for an acceleration, finds the correct vector ands then goes on to give just the modulus as the answer).
- 4. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular M or B mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are run together by the candidate, the earlier marks are implied and full credit must be given.

- 5. The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A and B marks are not given for 'correct' answers or results obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be agreed at the standardisation meeting.
- 6. For a partially correct **part** of a question, exhibit the detailed marks, e.g. M1, A0, *in the margin* at the point where the marks have been first earned. Please give sufficient detail to allow your marking to be understood. For a completely correct **part** of a question, only the total mark for that part need be given, *in the margin*. Do not use subtotals (underlined or otherwise). The question total should be ringed and placed in the margin at the end of the question. *This total MUST equal the sum of all the marks in the margin for that question* and should be entered against the question number in the question grid on the front of the script.
- 7. If a candidate's answer to a question is in more than one instalment, indicate the carried forward total at the end of one instalment by, for example, (3) and the brought forward total at the start of the next instalment by, for example, (3).
- 8. The total mark for the paper should be obtained (a) by adding all the unringed marks through the script (checking at the same time that all pages have been marked) and (b) by adding the question marks in the grid in reverse order. The two totals must, of course, tally, and the resulting figure should be written, ringed, on the front of the script.
- 9. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* get full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.
- 10. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a single candidate adopts a method which does not correspond to the marking scheme, award marks according to the spirit of the basic scheme; this will usually involve preserving the ratio **M**: **A** + **B**: **F** marks and any emphasis on accuracy specific to that question. If you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.
- 11. If a GDC is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- 12. If in any case the scheme operates with considerable unfairness, mark at discretion but please give a brief reason and initial the mark. *This discretion should be used very rarely, and only after consultation with your Team Leader*.
- 13. If there is any suspicion of cheating or copying, mark according to the scheme and enter the marks on the marksheet as usual. Send the script(s) to your Team Leader, as per OCR instructions. Notes concerning illness etc should be forwarded to OCR, but separately from the scripts or mark sheets. The scripts should be marked exactly as per the scheme, as any special circumstances that may have affected a candidate's performance are dealt with at a later stage.

14. Examiners are reminded of the *VITAL* importance of checking the accuracy of the addition of marks and of the transcriptions onto the marksheets; in particular that the marks are entered against the right candidates. Do not assume that the scripts are in the same order as the names on the marksheet. As detailed in §8 above, each Examiner must check the paper total, obtaining the same figure twice by different methods. The transcription to the marksheet should also be checked; ideally, the Checker should read out the candidate's name and mark from the marksheet, while the Examiner checks with the front of the script.

The Examiner has final responsibility for the accuracy of the mark recorded on the marksheet.

#### Tariffs and usage for common errors

# Note that these general instructions may be varied in individual questions or papers

#### Wrong value of g used

No penalty. E marks will be lost except when results agree to the accuracy required in the question.

#### Misread

Lose 1 mark for each consistent misread. FT except when trivialized. E marks lost unless, by chance, the given results are established by equivalent working.

#### Units

Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metre unless in a particular question all the lengths are km, when this would be assumed to be the unspecified unit.)

#### Accuracy

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

#### When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

#### When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f..

FT should be used so that only one mark is lost for each distinct error, except when for errors due to premature approximation which should be penalised only once in the examination.

#### Rules for crossed out and/or replaced work

If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

#### Some commonly used abbreviations

- AEF Any Equivalent Form (of answer or result) is equally acceptable
- AG Answer Given on the question paper (so extra care is needed in checking that the detailed working leading to the result is valid)
- BOD Benefit Of Doubt (allowed for work whose validity may not be absolutely plain)
- CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed
- FT Follow through
- ISW Ignore Subsequent Working.
- MR Misread
- NOS Used to indicate an allocation of marks that is Not On Scheme
- PA Premature Approximation (resulting in basically correct work that is numerically insufficiently accurate)
- SOS See Other Solution (the candidate makes a better attempt at the same question)
- SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
- WW Without Working
- WWW Without Wrong Working

		1	
1 (i)	$\frac{\mathrm{d}x}{\mathrm{d}t} = A\omega\cos\omega t$	B1	
	$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -A\omega^2 \sin \omega t$	M1	Obtaining second derivative
	$= -\omega^2 (A\sin \omega t) = -\omega^2 x$	E1	
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 = A^2 \omega^2 \cos^2 \omega t = A^2 \omega^2 (1 - \sin^2 \omega t)$	M1	Using $\cos^2 \omega t = 1 - \sin^2 \omega t$
	$= \omega^{2} (A^{2} - A^{2} \sin^{2} \omega t) = \omega^{2} (A^{2} - x^{2})$	E1 5	
(ii)	$1.2^2 = \omega^2 (A^2 - 0.7^2)$	M1	Using $v^2 = \omega^2 (A^2 - x^2)$
	$0.75^2 = \omega^2 (A^2 - 2^2)$	A1	Two correct equations (M0 if $x = 7.3$ used, etc)
	$\frac{A^2 - 0.49}{A^2 - 4} = \frac{1.2^2}{0.75^2}$	M1	Eliminating $\omega$ or Eliminating $A$
	$A^{2} - 0.49 = 2.56(A^{2} - 4)$ $9.75 = 1.56A^{2}$		or Substituting $A = 2.5$ into both equations
	$A^2 = 6.25$		
	Amplitude is 2.5 m	E1	Correctly shown
	$1.44 = \omega^2 (2.5^2 - 0.7^2)$		
	$\omega = 0.5$ Period is $\frac{2\pi}{\omega} = \frac{2\pi}{0.5}$	M1	Using $\frac{2\pi}{\alpha}$
	$\omega = 0.5$ = $4\pi = 12.6 \text{ s}$ (3 sf)	A1 6	$\omega$
(iii)	Maximum speed is $A\omega = 1.25 \text{ m s}^{-1}$	B1 <b>1</b>	ft only if greater than 1.2
(iv)	Magnitude is $0.5^2 \times 1.6$	M1	
	$= 0.4 \text{ m s}^{-2}$	A1	Accept -0.4
	Direction is upwards	B1 3	B0 for just 'towards centre'
(v)	$x = 2.5\sin(0.5t)$	B1	or $x = 2.5\cos(0.5t)$
	When $x = -2$ , $t = -1.855$ (or 10.71)		or $t = (\pm) 4.996$
	When $x = 1$ , $t = 0.823$ (or 13.39)		or $t = (\pm) 2.319$
	Time taken is $0.823 - (-1.855)$	M1	Correct strategy for finding time (must use radians)
	$= 2.68 \mathrm{s}$ (3 sf)	A1 3	(ft is $1.3388/\omega$ )

<b>A</b> ( ) (*)			
2(a)(i)	2	M1	For acceleration $\frac{u^2}{3.2}$
	$0.6 + 0.2 \times 9.8 = 0.2 \times \frac{u^2}{3.2}$	A1	3.2
	3.2	AI	
	Speed is 6.4 m s <sup>-1</sup>	A1	
			3
(ii)	(A)	M1	Equation involving KE and PE
	$\frac{1}{2}m(v^2-u^2) = m \times 9.8(3.2+3.2\cos 60^\circ)$	A1	
	$v^2 = 135.04$		
			2
	Radial component is $\frac{v^2}{3.2} = 42.2 \text{ m s}^{-2}$	A1	(ft is $29.4 + \frac{u^2}{3.2}$ )
	3.2		3.2
	Tangential component is $g \sin 60^{\circ}$	M1	M1A0 for $g \cos 60^{\circ}$
	$= 8.49 \mathrm{m  s^{-2}}$ (3 sf)	A1	M0 for $mg \sin 60^{\circ}$
	= 0.47 iii		If radial and tangential components
			are interchanged, withhold first A1
	(B)		
	$T - mg\cos 60^\circ = ma$	M1	Radial equation (three terms)
			(Allow M1 for $T - mg = ma$ )
			This M1 can be awarded in (A)
	$T - 0.2 \times 9.8 \cos 60^{\circ} = 0.2 \times 42.2$	A1	ft dependent on M1 for energy in (A)
	Tension is 9.42 N	A 1	
	Tension is 9.42 iv	711 040	SC If 60° with upward vertical, (A) M1A0A0 M1A1 (8.49)
			(A) MIA0A0 MIAI (8.49)
			(B) M1A1A1 (3.54)
(b)(i)	$T\cos 36^{\circ} + 0.75\sin 36^{\circ} = 0.2 \times 9.8$	M1	Resolving vertically (three terms)
(b)(i)			Allow sin/cos confusion, but both T and
	Tension is 1.88 N (3 sf)	A1	R must be resolved
			2
(ii)	Angular speed $\alpha = 2\pi$ (= 3.401)		Or $v = \frac{2\pi r}{1.8}$
	Angular speed $\omega = \frac{2\pi}{1.8}$ (= 3.491)	B1	$O(V - \frac{1.8}{1.8})$
	2	M1	Horiz eqn involving $r\omega^2$ or $v^2/r$
	$T\sin 36^{\circ} - 0.75\cos 36^{\circ} = 0.2 r \left(\frac{2\pi}{1.8}\right)^{2}$	A1	Equation for $r$ (or $l$ )
			, , ,
	r = 0.204		
	Length of string is $\frac{r}{\sin 36^{\circ}}$	M1	Dependent on previous M1
	= 0.347  m (3 sf)	A1 cao	_
			)

		1					
3 (i)	Elastic energy is $\frac{1}{2} \times \frac{573.3}{3.9} \times 0.9^2$	M1	Allow one error				
	= 59.535 J	A1	(Allow 60 A0 for 59)				
(ii)	Length of string at bottom is $2\sqrt{1.8^2 + 2.4^2}$ (= 6)	M1	Finding length of string (or half-string)				
	1 1 2/3.3 (2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	M1 B1B1	Equation involving EE and PE For change in EE and change in PE				
	324.135 - 59.535 = 17.64m						
	Mass is 15 kg	E1	5				
(iii)	Length of string is $2\sqrt{1.0^2 + 2.4^2} = 5.2$						
	550.0	M1	Finding tension (via Hooke's law)				
	Tension $T = \frac{573.3}{3.9} \times 1.3  (=191.1)$	A1					
	$2T\sin\alpha - mg = 2 \times 191.1 \times \frac{1.0}{2.6} - 15 \times 9.8$	M1	Finding vertical component of tension Give A1 for $T = 191.1$ obtained from				
	=147-147		resolving vertically				
	= 0, hence it is in equilibrium	E1	SC If 573.3 is used as stiffness: (i) M1A0 (ii) M1M1B0B1E0 (iii) M1A1 (745.29) M1E0				
(iv)	$[8\pi^2 h^3] = L^3, [8h^3 - ad^2] = L^3$		Condone $L^3/L^3 = 0$ , dimensionless				
	So $\frac{8\pi^2 h^3}{8h^3 - ad^2}$ is dimensionless	E1	But E0 for $\frac{L^3}{L^3 - L^3} = \frac{L^3}{0}$				
(v)		B1	For $[\lambda] = MLT^{-2}$				
	$T = M^{\alpha} L^{\beta} (M L T^{-2})^{\gamma}$						
	$\gamma = -\frac{1}{2}$	B1	If $\gamma$ is wrong but non-zero,				
	$\alpha + \gamma = 0$ , so $\alpha = \frac{1}{2}$	B1	give B1 ft for $\alpha = \beta = -\gamma$				
	$\beta + \gamma = 0$ , so $\beta = \frac{1}{2}$	B1	, ,				
			4				
(vi)	$a = 3.9$ , $\lambda = 573.3$ , $d = 4.8$ , $h = 2.6$ , $m = 15$	M1	Using formula with numerical at 0 at				
	$\sqrt{8-2L^3}$	M1	Using formula with numerical $\alpha$ , $\beta$ , $\gamma$ (must use the complete formula)				
	Period is $\sqrt{\frac{8\pi^2 h^3}{8h^3 - ad^2}} m^{\frac{1}{2}} a^{\frac{1}{2}} \lambda^{-\frac{1}{2}} = 1.67 \text{ s}$ (3 sf)	A1 cao	2				
	1		4				

4 (i)	c3 2	M1	
<b>4</b> (1)	Area is $\int_0^3 (x^2 + 5) dx$	1411	For $\int (x^2 + 5) dx$
	$= \left[ \frac{1}{3}x^3 + 5x \right]_0^3  (=24)$	A1	For $\frac{1}{3}x^3 + 5x$
	$\int xy  \mathrm{d}x = \int_0^3 (x^3 + 5x)  \mathrm{d}x$	M1	For $\int xy  dx$
	$= \left[ \frac{1}{4}x^4 + \frac{5}{2}x^2 \right]_0^3  (=\frac{171}{4})$	A1	For $\frac{1}{4}x^4 + \frac{5}{2}x^2$
	$\overline{x} = \frac{42.75}{24} = \frac{57}{32} = 1.78125$	A1	
	$\int \frac{1}{2} y^2 dx = \int_0^3 \frac{1}{2} (x^4 + 10x^2 + 25) dx$	M1	For $\int y^2 dx$
	$= \left[ \frac{1}{10}x^5 + \frac{5}{3}x^3 + \frac{25}{2}x \right]_0^3  (=106.8)$	A2	For $\frac{1}{10}x^5 + \frac{5}{3}x^3 + \frac{25}{2}x$
	$\overline{y} = \frac{106.8}{24} = \frac{89}{20} = 4.45$	A1 9	Give A1 for two terms correct
(ii)	Volume is $\int \pi x^2 dy = \int_5^{14} \pi (y - 5) dy$	M1	For $\int (y-5) dy$
	$= \pi \left[ \frac{1}{2} y^2 - 5y \right]_5^{14}  (=40.5\pi)$	A1	For $\left[\frac{1}{2}y^2 - 5y\right]_5^{14}$
	$\int \pi x^2 y  dy = \int_5^{14} \pi (y^2 - 5y)  dy$	M1	For $\int x^2 y  dx$
	$=\pi \left[ \frac{1}{3}y^3 - \frac{5}{2}y^2 \right]_5^{14}  (=445.5\pi)$	A1	For $\frac{1}{3}y^3 - \frac{5}{2}y^2$
	$\overline{y} = \frac{445.5\pi}{40.5\pi}$	M1	Dependent on previous M1M1
	=11	A1 6	
(iii)	Volume of whole cylinder is $\pi \times 3^2 \times 14 = 126\pi$		
	$126\pi \times 7 = 40.5\pi \times 11 + (126\pi - 40.5\pi) \times \overline{y}_A$	M1 A1	Using formula for composite body
	$\overline{y}_A = \frac{126\pi \times 7 - 40.5\pi \times 11}{126\pi - 40.5\pi}$		
	$= \frac{97}{19} = 5.105  (4 \text{ sf})$	A1 cao	

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# 4763: Mechanics 3

#### **General Comments**

The work on this paper was generally of a high standard and presented clearly. The marks were somewhat lower than last year, but even so about 40% of the candidates scored 60 marks or more (out of 72). The topic which caused the most difficulty was circular motion.

#### **Comments on Individual Questions**

1 (Simple harmonic motion)

This question was well answered and about a quarter of the candidates scored full marks.

- (i) Most candidates derived the two given results confidently. Some did not see how to use  $\cos^2 \omega t = 1 \sin^2 \omega t$  to obtain the second result, and there were a few sign errors in the differentiations.
- (ii) Almost all candidates knew that they should substitute into the formula  $v^2 = \omega^2 (A^2 x^2)$ , and this was often completed successfully. However, many took x to be the height above the sea-bed instead of the displacement from the centre of oscillation. Some omitted to find the period.
- (iii) The maximum speed was usually found correctly.
- (iv) The acceleration, and its direction, were usually given correctly. Here again the height 6.4 m was sometimes used instead of the displacement  $(-)1.6\,\mathrm{m}$ .
- (v) Most candidates had an appropriate displacement-time equation, but about half did not select an interval during which the ball is moving upwards. There were also a few working in degrees instead of radians.

#### 2 (Circular motion)

This was the worst answered question and only about 10% of candidates scored full marks.

- (a)(i) This was usually answered correctly. Sometimes there was a sign error in the equation of motion, and the mass was sometimes omitted.
- (a)(ii) The tangential component of acceleration ( $g \sin 60^\circ$ ) was usually given correctly, although quite a number gave it as  $mg \sin 60^\circ$ . Many candidates attempted to find the radial acceleration, and the tension in the string, without using the conservation of energy to find the speed first.
- (b)(i) A surprising number of candidates omitted the normal reaction when resolving vertically, or tried to resolve in the direction of the string.

(b)(ii) In this part most candidates did consider the horizontal components of both the tension and the normal reaction, and used the given period to find the angular velocity. Only a few obtained the correct length for the string, as this required a correct value for the tension and accurate work in this part. Some found the radius of the circle but did not proceed to find the length of the string.

#### 3 (Elasticity and dimensional analysis)

This question was quite well answered and about 20% of candidates scored full marks.

- (i) Almost all candidates calculated the elastic energy correctly. Just a few took 573.3 to be the stiffness instead of the modulus of elasticity.
- (ii) Although some treated the lowest point as a position of equilibrium rather than one of instantaneous rest, the majority considered elastic and gravitational potential energy and obtained the given mass convincingly.
- (iii) The work in this part was often less convincing; the tension was not always clearly stated, a factor 2 was sometimes missing, and the resolution of the tension in the vertical direction was often unclear.
- (iv) Almost every candidate could explain why the given expression is dimensionless.
- (v) The method for finding the powers was very well understood, and most candidates obtained the correct values. Common errors arose from having the wrong dimensions for  $\lambda$  (usually  $MT^{-2}$  instead of  $MLT^{-2}$ ) or the period (usually  $T^{-1}$  instead of T).
- (vi) This part was also well understood, and a good number obtained the correct answer. Some candidates omitted the dimensionless part and just evaluated  $m^{\alpha}a^{\beta}\lambda^{\gamma}$ .

#### 4 (Centres of mass)

About a quarter of the candidates scored full marks on this question.

- (i) The methods for finding the centre of mass of a lamina were very well understood and usually carried out accurately. A factor  $\frac{1}{2}$  was sometimes missing from the *y*-coordinate, and there were a few careless algebraic slips.
- (ii) Most candidates knew how to find the centre of mass of a solid of revolution. Not many errors were made in this part, but some used the wrong limits for the integration.
- (iii) Many candidates did not make any progress in this part. Those who put the pieces together to form a cylinder very often obtained the correct answer, but those who tried to find the centre of mass of S directly by integration very rarely earned any credit.



GCL IVIA	thematics (MEI)		May Mark		h		al		
4754/04	(CA) MELlipte direction to Advanced Mathematics	Daw	Max Mark	<b>a</b> 55	<b>b</b>	<b>c</b> 43	<b>d</b> 37	<b>e</b> 32	<b>u</b> 0
+/51/01	(C1) MEI Introduction to Advanced Mathematics	Raw UMS	72 100	55 80	70	43 60	50	32 40	0
1750/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	53	46	39	33	27	0
4/32/01	(C2) IVIET Concepts for Advanced Mathematics	UMS	100	80	70	60	50	40	0
1752/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	48	42	36	29	0
	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
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	(C4) MEI Applications of Advanced Mathematics (C4) MEI Applications of Advanced Mathematics	Raw	90	63	56	50	44	38	0
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	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
1758	(DE) MEI Differential Equations with Coursework  (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
		Raw	72	60	52	44	36	28	0
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762/01	(M2) MEI Mechanics 2	Raw	72	64	57	51	45	39	0
702/01	(MZ) MEN MEGNATIOS Z	UMS	100	80	70	60	50	40	0
763/01	(M3) MEI Mechanics 3	Raw	72	59	51	43	35	27	0
7 00/01	(MO) WEI WOOHAMOS O	UMS	100	80	70	60	50	40	0
764/01	(M4) MEI Mechanics 4	Raw	72	54	47	40	33	26	0
704/01	(NIT) NIET MEGNATIOS T	UMS	100	80	70	60	50	40	0
1766/01	(S1) MEI Statistics 1	Raw	72	53	45	38	31	24	0
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1767/01	(S2) MEI Statistics 2	Raw	72	60	53	46	39	33	0
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1768/01	(S3) MEI Statistics 3	Raw	72	56	49	42	35	28	0
77 00/01	(OO) INET OLUMBRIOS O	UMS	100	80	70	60	50	40	0
1769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
7 00/01	(OF) INEL OLUBOROS F	UMS	100	80	70	60	50	40	0
771/01	(D1) MEI Decision Mathematics 1	Raw	72	51	45	39	33	27	0
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772/01	(DZ) MEI Decision Watherlands Z	UMS	100	80	70	60	50	40	0
773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
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776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	62	55	49	43	36	0
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	(NM) MEI Numerical Methods with Coursework: Coursework  (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
1776	(NM) MEI Numerical Methods with Coursework  (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
777/01		Raw	72	55	47	39	32	25	0
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